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The eigenvalue problem and its relevance to the optimal configuration of electrodes for ultrasound actuators

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Abstract

Usually piezoelectric actuators operate in resonance mode, and to achieve the modal shape needed, it's necessary to obtain the exact dislocation of excitation zones. In order to use an actuator for ultrasonic devices, it must meet specific requirements for contact point movement, i.e., the trajectory of a contact point must have elliptical form. Changing geometrical parameters of dynamical structures can maximize effective work. Such kind of simulation leads to an unstable sequence of structural modal shapes, i.e., the structure of the same shape but different geometrical parameters has a different sequence of the modal shapes. Problem arises when we try to automate the actuator modelling process. Solution of the problem usually doesn't converge, and the numerical analysis becomes meaningless.

This paper presents a study of optimizing electrodes dislocation of ultrasonic actuators. The following conditions of optimization problem are considered: to unify excitation voltage forms, to achieve reverse motion and the maximum coefficient of efficiency. Finite element method modelling is performed in calculation process. The results of calculations for the piezoelectric drive are shown for two options of fixing conditions.

1. Introduction

A great number of ultrasonic devices such as positioning drives, micromanipulators, micropumps, transducers for measuring physical properties of materials, transmission of motion into vacuum chambers without power losses, converters, vibration concentrators, scanners, etc. are based on utilization of the particular resonance oscillations of a piezoelectric actuator. In most cases these devices simply lose their ability to operate if actuators vibrate in different frequency, because the modal shape of an actuator does not meet the requirements [1]. Modelling of the piezoelectric actuator can be divided into following steps: simplified analytical study, numerical simulation using finite element method (FEM), optimization of the actuator and development of the control scheme. Optimization problem has the following steps: optimization of dimensions of the actuator and optimization of configuration of the electrodes dislocation. Optimizing dimensions of the actuator, results from modal-shape analysis are used [2,3]. So it is

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important to know sequence of modal shapes, because it changes when dimensions of the actuator are changing. Unstable sequence of the modal shapes prevents automation of optimization process.

When piezoelectric actuator is tied up with particular resonant frequency and modal shape, optimal dislocation of excitation zones must be found. The optimal configuration of electrodes gives an opportunity to reduce the concentration of mechanical stresses and maximize amplitudes of vibrations. This fact is especially important for the multi-degrees of freedom (MDOF) ultrasonic actuators. In order to achieve the maximum coefficient of efficiency of piezoelectric actuators, the surface of an actuator must be fully covered with electrodes and excited only in corresponding zones. When FEM is used for simulations, precision of calculations for the excitation zone dislocations within the limits of the area of a finite element is achieved. Optimization of electrodes configuration also tied up with results from modal shape analysis [4].

This paper gives a study of eigenvalue problem and its relevance to optimization of electrodes configuration of piezoelectric actuator.

2. Problem definition

Since the analysis of a multidimensional piezoelectric actuator cannot be performed without considering the vibration device, most often the problems of piezoelectric actuator research are solved in an integral fashion taking into account the whole device. In order to use actuator for ultrasonic devices, it must meet specific requirements for contact point movement, i.e., the trajectory of contact point must have elliptical form. This could be achieved when MDOF oscillations are excited. The shape and optimal location of the electrodes on the surface of an actuator have great importance to vibration mode of an actuator. Using different configurations of electrodes, vibration of the main and higher resonance modes of an ultrasonic actuator could be achieved [5]. In case of the optimal electrode configuration, needless harmonics of an actuator could be eliminated and the concentration of mechanical stresses could be reduced. These facts are very important in case of MDOF oscillations of an actuator that is analysed in this paper.

The formal algorithm for solving the problem looks as follows: Fig. 1. In the case under consideration, at the first stage the geometrical parameters of a piezoelectric actuator are changed. At the second stage a matrix of eigenvalues is formed, whose every column describes a corresponding modal shape; the first column describes the first modal shape, the second column—the second shape and so on. While changing the geometrical parameters of a piezoelectric actuator the change in the modal shape sequence has been observed.

Using the technical oscillation theory of a beam the longitudinal oscillations are found by solving the second-order differential equation [6]:

$$m(x)\frac{\partial^2 \xi}{\partial^2 t} - \frac{\partial}{\partial x}\left(Es\frac{\partial \xi}{\partial x}\right) = 0, \tag{1}$$

where s is cross-section area.

Longitudinal oscillations of the beam can be expressed as follows [6]:

$$\omega_k = \frac{k}{2l}\sqrt{\frac{E}{\rho}}, \tag{2}$$

E is the Young's modulus; k is the mode number of the longitudinal oscillations; l is the length of a beam; and ρ is mass density.

Flexural oscillations of a beam are found by solving the second-order differential equation [6]:

$$\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 \xi_{\text{len}}}{\partial x^2}\right) + \rho S\frac{\partial^2 \xi_{\text{len}}}{\partial^2 t} = 0. \tag{3}$$

Flexural oscillations of a beam are described by the expression [6]:

$$\omega_n = \frac{\pi h(n+0.5)^2}{4l^2}\sqrt{\frac{E}{3\rho}}, \tag{4}$$

h is the height of a beam; and n is the mode number of flexural oscillations.

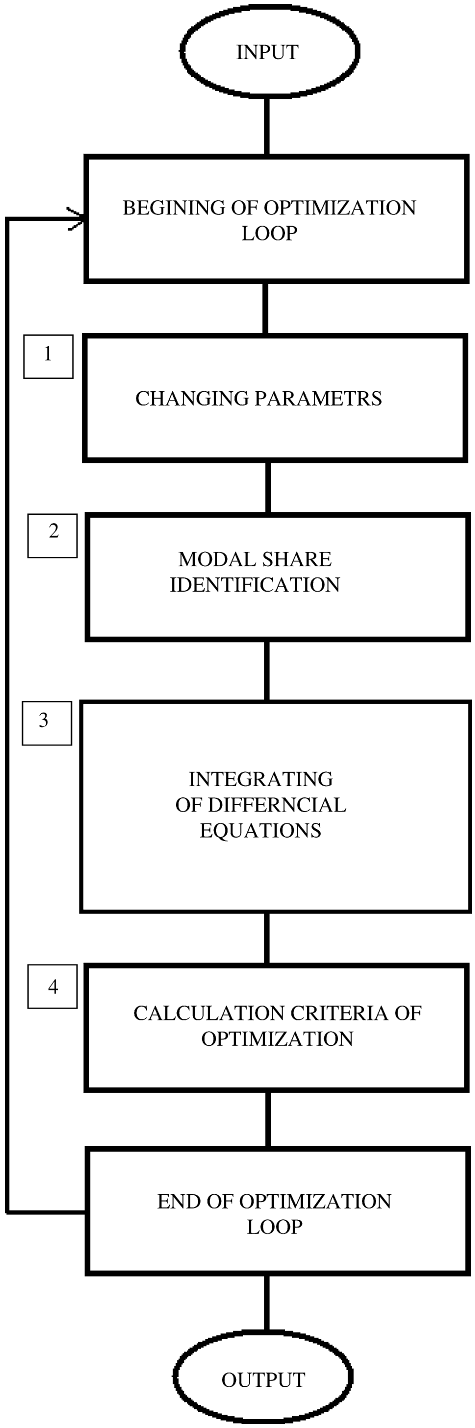

Fig. 1. The structure of the general calculation algorithm.

If certain values of k and n are defined, then h/l ratio of the beam could be calculated. From Eqs. (2) and (4) the following equation could be obtained:

$$\frac{h}{l} = \frac{2\sqrt{3}k}{\pi(n+0.5)^2}. \tag{5}$$

As an example:

$$k = 1; \quad n = 2 \Rightarrow \frac{h}{l} = 0.1765, \tag{6}$$

but h/l ratio could be changed, for example, increasing or reducing the height of a beam. In this case k value remains the same, but n value changes. This means that the sequence of modal shapes changes when the geometrical parameters of a beam vary. Calculations of MDOF piezoelectric actuators, of course, have the same problem, but is very difficult or even impossible to solve it analytically. Modal shape sequence exchanges often cause fatal errors in the calculation process as well as unexpected errors in results [3]. And it means that at the second stage of solving the problem an incorrect value of ω_k (natural frequency) can be chosen. This problem is also important for solving optimization problems because calculations are usually related not only to a particular natural frequency of a piezoactuator but also to its modal shape. Therefore, it is necessary to recognize numerically modal shapes and to determine their location (column numbers) in the matrix of modal shapes for the construction model. For solving this problem we will apply the energy solution method analysed in Section 3.

Having chosen the suitable value of ω_k we can further solve the problem of an optimal configuration of multidimensional piezoactuator electrodes. The shape and optimal location of the electrodes on the surface of an actuator have great importance to the vibration mode of an actuator. Using different configurations of electrodes, vibration of main and higher resonance modes of a piezoelectric actuator could be achieved. In case of the optimal electrodes configuration, needless harmonics of an actuator could be eliminated and the concentration of mechanical stresses could be reduced as well. These facts are very important in case of MDOF oscillations of an actuator that are analysed in Section 5 of this paper.

3. An algorithm for modal shape identification

When the modal frequencies analysis of MDOF actuators is done using FEM, dominating components of the oscillations can be found referring to the energetic method of the oscillation analysis, because amplitudes raised to the second power are proportional to the energy of the oscillations. In that way the ratios (dominating coefficients) of the components of amplitudes in all directions can be found and the direction with the maximum of amplitudes can be defined. Let's calculate the following sum [2,3]:

$$S_p^b = \sum_{i=1}^{r} (A_{ip}^b)^2, \quad r = \frac{q}{p}, \tag{7}$$

S_p^b is the sum of oscillations amplitudes in p direction; A_{ip}^b is the amplitude of oscillations of i element of modal shape vector in p direction; r is the size of modal shape vector for p coordinate; q is the degrees of freedom (dof) of actuator model; p is the number of dof in the node.

The dominating coefficients of the model can be expressed as follows [2,3]:

$$m_{jk}^b = \frac{S_j^b}{S_k^b}, \quad j \neq k. \tag{8}$$

The physical meaning of dominating coefficients is the ratios between different oscillation energy components in the directions of coordinate axes. The sum S_k^b defines the energy of the oscillation of the b natural frequency in k direction and the dominating coefficient m_{jk}^b defines the relation of the oscillation energies in i and j directions of b natural frequency. Dominating coefficients have the following characteristic [2,3]:

$$m_{kj}^b = \frac{1}{m_{jk}^b}, \quad j \neq k. \tag{9}$$

Based on dominating coefficients we could determinate the type of dominating oscillations and also define the level of correlation of MDOF oscillations as follows:

$$m_{kj}^{b} = \tau \Rightarrow lg\,\tau = 1, 2, 3, \ldots, \quad (10)$$

where τ is the variable, defining correlation of the frequencies.

Dominating coefficients is a means to define the type of structure oscillation and to sort modal shapes by dominating type of the oscillations for example longitudinal, flexural, and torsional. In order to finally identify modal shape additional criteria must be used. We offer to use the number of node points or the number of node lines of the modal shape for regular mechanical structures as additional criteria (Fig. 2).

During calculations the number of nodes points or lines could be found by referring to the alternation of the sign of the oscillation amplitude around the equilibrium level (Fig. 2). The exchanges in the modal shape sequence are a general case problem concerning not only piezoelectric actuators, but also all mechanical structures.

4. Numerical investigation and results

Based on the algorithm of modal shape identification, numerical testing was carried out. A two-dimensional (2D) plane structure, a cylinder and an irregular structure were tested (Fig. 3).

The aim of the numerical analysis was to determine the dependencies of modal shape sequence on geometrical parameters of the structure. Exchanges of modal shapes of the second and third natural frequencies of a 2D plane structure are shown in Fig. 4. Dominating coefficients m_{12} represent the ratio of oscillation energy in x and y directions of coordinate axes, respectively. The analysis of modal shapes of a cylinder must be done taking into account the ration of cylinder wall thickness and internal radius. When the ratio of the wall thickness and radius is $0.25 < h/R_{\text{vid}} < 0.5$, the transition from thin-layered cylinder modal shapes to thick-layered cylinder modal shapes happens. This does not depend on cylinder boundary conditions. Dependences of the dominating coefficient m_{13} of cylinder third natural frequency (Fig. 5) show exchanges of modal shapes.

Investigation of dominating coefficients of an irregular structure was done separately for each structural component. Most irregular structures can be divided into a set of regular structures, so analysis of dominating coefficients of a set of regular structures is more accurate and more sensitive to the changes of modal shape of a structure. The dependence of the dominating coefficient m_{12} of horizontal and vertical bars (Fig. 6) exactly

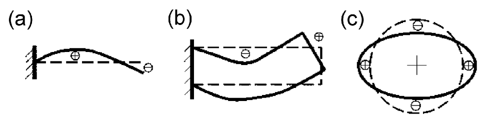

Fig. 2. Modal shape identification of the regular mechanical structures: (a) beam, (b) plate, and (c) cylinder.

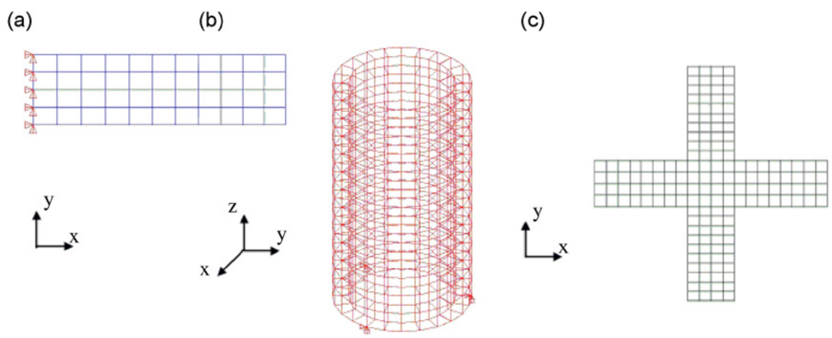

Fig. 3. FEM models of the analysed mechanical structures: (a) two-dimensional plane structure; (b) cylinder; and (c) irregular structure.

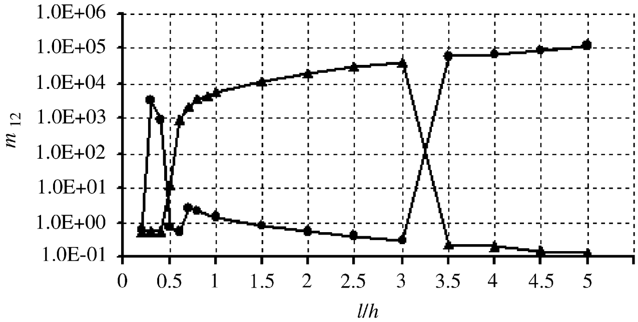

Fig. 4. The dependence of the dominating coefficient m_{12} on the length and height ratio l/h of a two-dimensional plane structure for the second and third natural frequency when the left side nodes are fixed. –●–2n.d., –▲–3n.d.

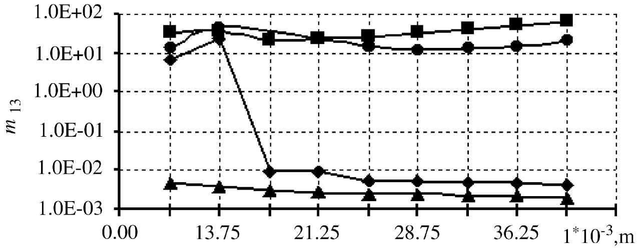

Fig. 5. The dependence of the dominating coefficient m_{13} of cylinder length for the third natural frequency. Wall thickness of a cylinder is 0.0025 m. –▲–$R = 0.005$ m; –◆–$R = 0.0078$ m;–■– $R = 0.01$ m; –●– $R = 0.015$ m.

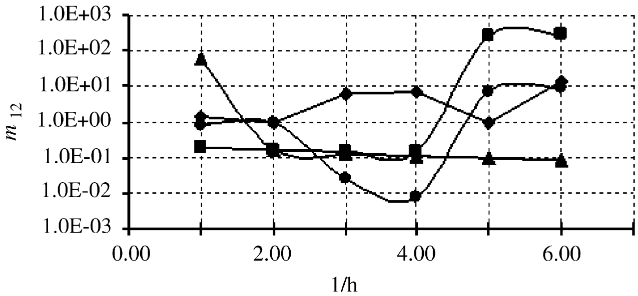

Fig. 6. The dependence of the dominating coefficient m_{12} of an irregular structure. –▲–1n.f.hor.; –◆– 1n.f.vert.; –■– 2n.f.hor.; –●–2n.f.vert.

shows exchanges of modal shapes of an irregular structure depending on the length and height ratio of the vertical bar.

When an irregular structure, e.g., a plate with an aperture, is in consideration, dominated coefficients are distributed differently then usual depending on the ratio l/h. Therefore, dominating coefficients could be applied more widely. We could apply them in the process of fault identification.

5. Optimization analysis

In order to obtain the optimal electrodes configuration of an actuator, the following requirements of a piezoelectric system have to be met: to use unified generator of electric signal, to achieve reverse motion and the maximize efficiency. Realization of the first requirement means that the source of current has to generate voltage of stable frequency, amplitude and phase in order to simplify construction of the machine, second—that reverse motion must be achieved by changing the polarity of the electric flux and third—that the actuator must be fully covered with the electrodes. When all aforementioned conditions are achieved, any mode shape of the ultrasonic actuator could be obtained by changing polarity of the voltage, supplied to a particular electrode. In order to find the sign of the polarity corresponding to a certain modal shape, the comparisons between the directions of the vector of amplitude of the equivalent mechanical force and a particular eigenvector of the actuator must be made. If the direction of the displacement eigenvector of the piezoelectric actuator and the vector of amplitudes of the equivalent mechanical force is the same or similar, i.e., the angle is within the limits $[-\pi/2; +\pi/2]$, then the polarity of the voltage, supplied to a particular electrode has the initial sign. In the other case, the sign of polarity must be changed. So depending on the angle of aforementioned vectors, the polarity of voltage is defined.

The basic dynamic FEM equation of motion for piezoelectric transducer that is fully covered with electrodes can be expressed as

$$\mathbf{M}\ddot{\boldsymbol{\delta}} + \mathbf{C}\dot{\boldsymbol{\delta}} + \mathbf{K}\boldsymbol{\delta} - \mathbf{T}\varphi = \mathbf{F}, \quad \mathbf{T}^{\mathrm{T}}\boldsymbol{\delta} + \mathbf{S}\varphi = \mathbf{Q}, \tag{11}$$

where **M**, **K**, **T**, **S**, **C** are the matrices of mass, stiffness, electro elasticity, capacity and damping, respectively; $\boldsymbol{\delta}$,φ,**F**,**Q** are the vectors of nodes displacements, potentials and external mechanical forces, charges coupled on the electrodes respectively.

The operating frequency of the ultrasonic actuator is close to the resonance, so the excitation voltage must have the same frequency [4]:

$$\mathbf{M}\ddot{\boldsymbol{\delta}} + \mathbf{K}\boldsymbol{\delta} - \mathbf{T}\varphi = 0, \quad \mathbf{T}^{\mathrm{T}}\boldsymbol{\delta} + \mathbf{S}\varphi = 0. \tag{12}$$

The natural frequency and normalized displacement eigenvectors are derived from the modal solution of the piezoelectric system and used in further optimization analysis.

Because of the first objective of optimization to unify excitation voltage, the potential of electrodes of all piezoelements must be equal as shown in equation [5]:

$$\varphi^e = \mathbf{U}^e \mathrm{sg}^e \sin \omega_k t, \quad \mathrm{sg}^e = \begin{cases} +1, \\ -1, \end{cases} \tag{13}$$

where $\mathbf{U}^e$, sg^e, ω_k are the vector of element excitation voltage amplitude, modified sign function and k is resonance frequency, respectively. When the vector of the external mechanical forces is set to zero in Eq. (11), we obtain the following equations:

$$\mathbf{M}\ddot{\boldsymbol{\delta}} + \mathbf{C}\dot{\boldsymbol{\delta}} + \mathbf{K}\boldsymbol{\delta} = -\mathbf{F}, \tag{14}$$

Here,

$$\mathbf{F} = \sum_e \mathbf{L}^e\mathbf{T}^e\mathbf{U}^e \mathrm{sg}^e \sin \omega_k t = \sum_e \mathbf{F}^e \sin \omega_k t, \tag{15}$$

where **F** is the vector of external equivalent mechanical forces; $\mathbf{F}^e$ is the vector of amplitude of external equivalent mechanical force at the nodes of a finite element in a global coordinate system; $\mathbf{T}^e$ is the matrix of electroelasticity of a finite element; $\mathbf{L}^e$ is the matrix of transformation between local and global element coordinates.

Here,

$$\mathbf{F}^e = \mathbf{L}^e\mathbf{T}^e\mathbf{U}^e sg^e. \tag{16}$$

The solution of the basic dynamic FEM equation of motion for a piezoelectric actuator could be written in the following form:

$$\boldsymbol{\delta} = \boldsymbol{\Delta}_0 z(t), \tag{17}$$

where $\boldsymbol{\Delta_0}$ is normalized eigenvectors; $z(t)$ is the coefficient of proportionality.

Here,

$$\boldsymbol{\Delta_0} = [\boldsymbol{\delta}_{01}, \boldsymbol{\delta}_{02}, \ldots, \boldsymbol{\delta}_{0n}]. \tag{18}$$

The coefficient of proportionality could be obtained from equation:

$$\ddot{z}_i + 2\omega_i h_i \dot{z}_i + \omega_i^2 z_i = -\boldsymbol{\delta}_{0i}^{\mathrm{T}} \mathbf{F}, \quad i = 1, 2, \ldots, n. \tag{19}$$

Solving Eq. (19) we could calculate coefficient $z_k(t)$, that corresponds to k natural frequency of the actuator.

Now let us analyse the effective work of the external forces when electrodes of an actuator are excited. Referring to the third requirement of optimization problem to obtain the maximum of the coefficient of efficiency, the effective work of an actuator A_k^{ef} must be maximized. The average effective work, corresponding to the k natural frequency could be obtained as follows:

$$\max\ A_k^{\mathrm{ef}} = \frac{\omega_k}{2\pi} \int_0^{2\pi/\omega_k} \boldsymbol{\delta}_{0k}^{\mathrm{T}} \mathbf{F} z_k(t)\,\mathrm{d}t. \tag{20}$$

If we insert Eq. (14) into Eq. (19), the following expression of the effective work could be obtained:

$$\max\ A_k^{\mathrm{ef}} = \frac{\omega_k}{2\pi} \int_0^{2\pi/\omega_k} \sum_e \boldsymbol{\delta}_{0k}^{\mathrm{T}} \mathbf{F}^e \sin\,\omega_k t z_k(t)\,\mathrm{d}t. \tag{21}$$

Eq. (20) could be rewritten in the form:

$$\max\ A_k^{\mathrm{ef}} = \sum_e \boldsymbol{\delta}_{0k}^{\mathrm{T}} \mathbf{F}^e P_k = \sum_e |\boldsymbol{\delta}_0|_k |\mathbf{F}^e| \cos\,\gamma_k^e P_k, \tag{22}$$

Here,

$$P_k = \frac{\omega_k}{2\pi} \int_0^{2\pi/\omega_k} \sin\,(\omega_k t) z_k(t)\,\mathrm{d}t, \tag{23}$$

Based on Eq. (23) we can see that P_k is time-independent and referring to particular conditions $\boldsymbol{\delta}_{0k}$ is constant also, so only the product of the vector of the amplitude of the equivalent mechanical force and the eigenvector of an ultrasonic actuator must be maximized. In order to achieve the maximum value, directions of the vectors must be same or similar, i.e., $\cos\gamma_k^e$ is the cosine of the angle of aforementioned vectors must be maximized.

$$\cos\,\gamma_k^e = \frac{\boldsymbol{\delta}_{0k}^{\mathrm{T}} \mathbf{F}^e}{|\boldsymbol{\delta}_0|_k |\mathbf{F}^e|}. \tag{24}$$

As we can see from Eq. (24), the sign of $\cos\gamma_i^e$ depends only on the direction of the vector of the equivalent mechanical forces in a finite element. Referring to Eq. (16), the direction of the vector could be changed by changing the polarity of the voltage. Based on Eq. (13), the polarity of the voltage supplied to the element depends on the sg_j. The maximum of oscillation of the amplitude, depending on a certain modal shape of the piezodrive, is achieved when directions of the eigenvector and the vector of amplitudes of the equivalent mechanical force are similar, i.e., when value of sg_i is identical to the sign of $\cos\gamma_k^e$.

6. Processing and results

Calculations were carried out with a 2D piezoelectric actuator. The vector of actuator polarization is perpendicular to the paper plane, and all the structural dof of central nodes of the actuator are constrained. Initial values of electrical potential on electrodes are set to zero. The results of calculations $\cos\gamma_k^e$ are given in the centre of gravity of the element. Based on the sign and values of $\cos\gamma_k^e$, few zones of electrode location could be obtained (Fig. 7). Zones with different polarity are separated with a bold line. For comparison, in Fig. 8 we can see the configuration of electrodes for the same type of actuators created by the intuition of an engineer. These kinds of actuators are used in ultrasonic motors with rotational and linear motion. Values of $\cos\,\gamma_k^e$ are given in the centre of gravity of the elements. The vector of polarization is perpendicular to the paper plane.

The configuration of electrodes of a piezoelectric actuator according to engineer's intuition Fig. 8 created for the same type of actuators as shown in Fig. 7, respectively. The vector of polarization is perpendicular to the paper plane.

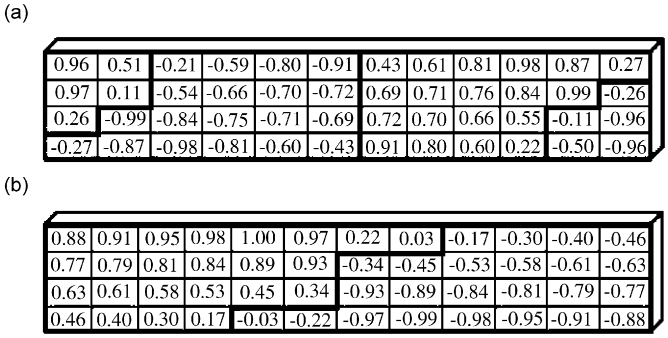

Fig. 7. The excitation zone configuration of a piezoelectric actuator: (a) the first modal shape, and (b) the second modal shape.

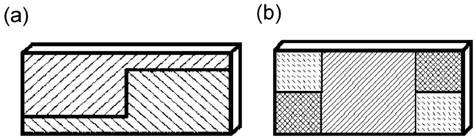

Fig. 8. The configuration of electrodes of a piezoelectric actuator according to engineer's intuition created for the same type of actuators as shown in Fig. 7, respectively.

7. Conclusions

Optimization of dimensions and electrodes location of piezoelectric actuators can proceed based on the results of eigenvalue problems, and identification of the modal shape sequence is a necessary step in order to automate optimization problem solving. The algorithm for modal shape identification must be used as an additional stage of the usual optimization algorithm.

The numerical algorithm of the excitation zone configuration for a MDOF piezoelectric actuator has been proposed. This algorithm is based on FEM and its accuracy of calculations depends on the limits of the area of a finite element. Using this optimization algorithm, the problem of durability and reliability of actuators could be solved.

References

[1] R. Bansevicius, R. Barauskas, G. Kulvietis, K. Ragulskis, *Vibromotors for Precision Microrobots*, Hemisphere Publishing Corporation, USA, 1988, p. 287.

[2] R. Bansevicius, G. Kulvietis, D. Mazeika, Design of the multicomponent ultrasonic drives based on mode-frequency analysis, *Mechanika N* 5 (20) (1999) 23–27.

[3] R. Bansevicius, G. Kulvietis, D. Mazeika, Numerical identification of modal shapes sequence of multicomponent piezoelectric actuators, *Journal of Vibroengineering* 2 (4) (2000).

[4] R. Bansevicius, G. Kulvietis, D. Mazeika, Optimisation of electrodes dislocation of multicomponent piezoelectric actuators, *Tenth World Congress on the Theory of Machine and Mechanisms*, Oulu, Finland, June 20–24, 1999.

[5] R. Bansevicius, G. Kulvietis, D. Mazeika, Optimal electrodes configuration of piezodrives, *Solid Mechanics and its Applications*, Vol. 73, Kluwer Academic Publishers, Dordrecht, 1998, pp. 77–83.

[6] Вибрации в технике. Подредакцией В.В. Волотина. Т 1. Москва: Машиностроение, 1978, 353с.